Chapter 5: Locus and Straight Line

EXERCISE 5.1 [PAGE 67]

Exercise 5.1 | Q 1 | Page 67

If A(1, 3) and B(2, 1) are points, find the equation of the locus of point P such that PA = PB.

SOLUTION

Let P(x, y) be any point on the required locus.
Given, A(1, 3) and B(2, 1).
PA = PB
$\therefore PA^2 = PB^2$
$\therefore (x - 1)^2 + (y - 3)^2 = (x - 2)^2 + (y - 1)^2$
$\therefore x^2 - 2x + 1 + y^2 - 6y + 9 = x^2 - 4x + 4 + y^2 - 2y + 1$
$\therefore -2x - 6y + 10 = -4x - 2y + 5$
$\therefore 2x - 4y + 5 = 0$
$\therefore$ The required equation of locus is $2x - 4y + 5 = 0$.

Exercise 5.1 | Q 2 | Page 67

A(– 5, 2) and B(4, 1). Find the equation of the locus of point P, which is equidistant from A and B.

SOLUTION

Let P(x, y) be any point on the required locus.
P is equidistant from A(– 5, 2) and B(4, 1).
$\therefore PA = PB$
$\therefore PA^2 = PB^2$
$\therefore (x + 5)^2 + (y - 2)^2 = (x - 4)^2 + (y - 1)^2$
$\therefore x^2 + 10x + 25 + y^2 - 4y + 4 = x^2 - 8x + 16 + y^2 - 2y + 1$
$\therefore 10x - 4y + 29 = - 8x - 2y + 17$
$\therefore 18x - 2y + 12 = 0$
$\therefore 9x - y + 6 = 0$
$\therefore$ The required equation of locus is $9x - y + 6 = 0$.

Exercise 5.1 | Q 3 | Page 67

If A(2, 0) and B(0, 3) are two points, find the equation of the locus of point P such that AP = 2BP.

SOLUTION

Let P(x, y) be any point on the required locus.
Given, A(2, 0), B(0, 3) and
AP = 2BP
$\therefore AP^2 = 4BP^2$
$\therefore (x - 2)^2 + (y - 0)^2 = 4[(x - 0)^2 + (y - 3)^2]$
$\therefore x^2 - 4x + 4 + y^2 = 4(x^2 + y^2 - 6y + 9)$
$\therefore x^2 - 4x + 4 + y^2 = 4x^2 + 4y^2 - 24y + 36$
$\therefore 3x^2 + 3y^2 + 4x - 24y + 32 = 0$
$\therefore$ The required equation of locus is
$3x^2 + 3y^2 + 4x - 24y + 32 = 0$

Exercise 5.1 | Q 4 | Page 67

If A(4, 1) and B(5, 4), find the equation of the locus of point P if $PA^2 = 3PB^2$.

SOLUTION

Let P(x, y) be any point on the required locus.
Given, A(4, 1), B(5, 4) and
$PA^2 = 3PB^2$
$\therefore (x - 4)^2 + (y - 1)^2 = 3[(x - 5)^2 + (y - 4)^2]$
$\therefore x^2 - 8x + 16 + y^2 - 2y + 1 = 3(x^2 - 10x + 25 + y^2 - 8y + 16)$
$\therefore x^2 - 8x + y^2 - 2y + 17 = 3x^2 - 30x + 75 + 3y^2 - 24y + 48$
$\therefore 2x^2 + 2y^2 - 22x - 22y + 106 = 0$
$\therefore x^2 + y^2 - 11x - 11y + 53 = 0$
$\therefore$ The required equation of locus is
$x^2 + y^2 - 11x - 11y + 53 = 0$.

Exercise 5.1 | Q 5 | Page 67

A(2, 4) and B(5, 8), find the equation of the locus of point P such that $PA^2 - PB^2 = 13$.

SOLUTION

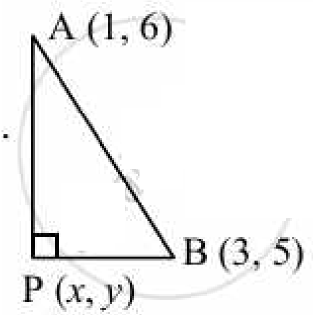
Let P(x, y) be any point on the required locus.
Given, A(2, 4), B(5, 8) and
$PA^2 - PB^2 = 13$
$\therefore [(x - 2)^2 + (y - 4)^2] - [(x - 5)^2 + (y - 8)^2] = 13$
$\therefore (x^2 - 4x + 4 + y^2 - 8y + 16) - (x^2 - 10x + 25 + y^2 - 16y + 64) = 13$
$\therefore 6x + 8y - 69 = 13$
$\therefore 6x + 8y - 82 = 0$
$\therefore 3x + 4y - 41 = 0$
$\therefore$ The required equation of locus is
$3x + 4y - 41 = 0$.

Exercise 5.1 | Q 6 | Page 67

A(1, 6) and B(3, 5), find the equation of the locus of point P such that segment AB subtends right angle at P. (∠APB = 90°)

SOLUTION

Let P(x, y) be any point on the required locus.
Given, A(1, 6) and B(3, 5),
(∠APB = 90°)
∴ ΔAPB = 90°
∴ ΔAPB is a right angled triangle.
By Pythagoras theorem,
$AP^2 + PB^2 = AB^2$



∴ $[(x - 1)^2 + (y - 6)^2] + [(x - 3)^2 + (y - 5)^2] = (1 - 3)^2 + (6 - 5)^2$
∴ $x^2 - 2x + 1 + y^2 - 12y + 36 + x^2 - 6x + 9 x y^2 - 10y + 25 = 4 + 1$
∴ $2x^3 + 2y^2 - 8x - 22y + 66 = 0$
∴ $x^2 + y^2 - 4x - 11y + 33 = 0$
∴ The required equation of locus is
$x^2 + y^2 - 4x - 11y + 33 = 0$.

Exercise 5.1 | Q 7.1 | Page 67

If the origin is shifted to the point O'(2, 3), the axes remaining parallel to the original axes, find the new co-ordinates of the points A(1, 3)

SOLUTION

Origin is shifted to (2, 3) = (h, k)
Let the new co-ordinates be (X, Y).
∴ x = X + h and y = Y + k
∴ x = X + 2 and y = Y + 3 …(i)
Given, A(x, y) = A(1, 3)
x = X + 2 and y = Y + 3 ...[From (i)]
∴ 1 = X + 2 and 3 = Y + 3

∴ $X = -1$ and $Y = 0$
∴ the new co-ordinates of point A are $(-1, 0)$

Exercise 5.1 | Q 7.2 | Page 67

If the origin is shifted to the point O'(2, 3), the axes remaining parallel to the original axes, find the new co-ordinates of the points B(2, 5)

SOLUTION

Origin is shifted to $(2, 3) = (h, k)$
Let the new co-ordinates be (X, Y).
∴ $x = X + h$ and $y = Y + k$
∴ $x = X + 2$ and $y = Y + 3$ …(i)
Given, $B(x, y) = B(2, 5)$
$x = X + 2$ and $y = Y + 3$ ..[From (i)]
∴ $2 = X + 2$ and $5 = Y + 3$
∴ $X = 0$ and $Y = 2$
∴ the new co-ordinates of point B are $(0, 2)$.

Exercise 5.1 | Q 8.1 | Page 67

If the origin is shifted to the point O'(1, 3), the axes remaining parallel to the original axes, find the old co-ordinates of the points C(5, 4)

SOLUTION

Origin is shifted to $(1, 3) = (h, k)$
Let the new co-ordinates be (X, Y).
$x = X + h$ and $y = Y + k$
∴ $x = X + 1$ and $y = Y + 3$ ...(i)
Given, $C(X, Y) = C(5, 4)$
$x = X + 1$ and $y = Y + 3$ …[From (i)]
∴ $x = 5 + 1 = 6$ and $y = 4 + 3 = 7$
∴ the old co-ordinates of point C are $(6, 7)$.

Exercise 5.1 | Q 8.2 | Page 67

If the origin is shifted to the point O'(1, 3), the axes remaining parallel to the original axes, find the old co-ordinates of the points D(3, 3)

SOLUTION

Origin is shifted to $(1, 3) = (h, k)$
Let the new co-ordinates be (X, Y).
$x = X + h$ and $y = Y + k$
∴ $x = X + 1$ and $y = Y + 3$ ...(i)
Given, $D(X, Y) = D(3, 3)$
$x = X + 1$ and $y = Y + 3$ …[From (i)]

∴ $x = 3 + 1 = 4$ and $y = 3 + 3 = 6$

∴ the old co-ordinates of point D are (4, 6).

Exercise 5.1 | Q 9 | Page 67

If the co-ordinates (5, 14) change to (8, 3) by shift of origin, find the co-ordinates of the point, where the origin is shifted.

SOLUTION

Let the origin be shifted to (h, k).

Given, (x, y) = (5, 14), (X, Y) = (8, 3)

Since, $x = X + h$ and $y = Y + k$

∴ $5 = 8 + h$ and $14 = 3 + k$

∴ $h = -3$ and $k = 11$

∴ the co-ordinates of the point, where the origin is shifted are (– 3, 11).

Exercise 5.1 | Q 10.1 | Page 67

Obtain the new equations of the following loci if the origin is shifted to the point O'(2, 2), the direction of axes remaining the same: $3x - y + 2 = 0$

SOLUTION

Given, (h, k) = (2, 2)

Let (X, Y) be the new co-ordinates of the point (x, y).

∴ $x = X + h$ and $y = Y + k$

∴ $x = X + 2$ and $y = Y + 2$

Substituting the values of x and y in the equation $3x - y + 2 = 0$, we get

$3(X + 2) - (Y + 2) + 2 = 0$

∴ $3X + 6 - Y - 2 + 2 = 0$

∴ $3X - Y + 6 = 0$, which is the new equation of locus.

Exercise 5.1 | Q 10.2 | Page 67

Obtain the new equations of the following loci if the origin is shifted to the point O'(2, 2), the direction of axes remaining the same: $x^2 + y^2 - 3x = 7$

SOLUTION

Given, (h, k) = (2, 2)

Let (X, Y) be the new co-ordinates of the point (x, y).

∴ $x = X + h$ and $y = Y + k$

∴ $x = X + 2$ and $y = Y + 2$

Substituting the values of x and y in the equation $x2 + y^2 - 3x = 7$, we get

$(X + 2)^2 + (Y + 2)^2 - 3(X + 2) = 7$

∴ $X^2 + 4X + 4 + Y^2 + 4Y + 4 - 3X - 6 = 7$

∴ $X^2 + Y^2 + X + 4Y - 5 = 0$, which is the new equation of locus.

Exercise 5.1 | Q 10.3 | Page 67

Obtain the new equations of the following loci if the origin is shifted to the point O'(2, 2), the direction of axes remaining the same: $xy - 2x - 2y + 4 = 0$

SOLUTION

Given, (h, k) = (2, 2)
Let (X, Y) be the new co-ordinates of the point (x, y).
∴ x = X + h and y = Y + k
∴ x = X + 2 and y = Y + 2
Substituting the values of x and y in the equation $xy - 2x - 2y + 4 = 0$, we get
$(X + 2)(Y + 2) - 2(X + 2) - 2(Y + 2) + 4 = 0$
∴ $XY + 2X + 2Y + 4 - 2X - 4 - 2Y - 4 + 4 = 0$
∴ XY = 0, which is the new equation of locus.

EXERCISE 5.2 [PAGES 69 - 70]

Exercise 5.2 | Q 1.1 | Page 69

Find the slope of the following lines which pass through the point: (2, – 1), (4, 3)

SOLUTION

Let A = (x_1, y_1) = (2, – 1) and B = (x_2, y_2) = (4, 3).

Slope of line AB = $\dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{3 - (-1)}{4 - 2}$

$= \dfrac{4}{2}$

= 2.

Exercise 5.2 | Q 1.2 | Page 69

Find the slope of the following lines which pass through the point: (– 2, 3), (5, 7)

SOLUTION

Let C = (x_1, y_1) = (– 2, 3) and D = (x_2, y_2) = (5, 7).

Slope of line CD = $\frac{y_2 - y_1}{x_2 - x_1}$

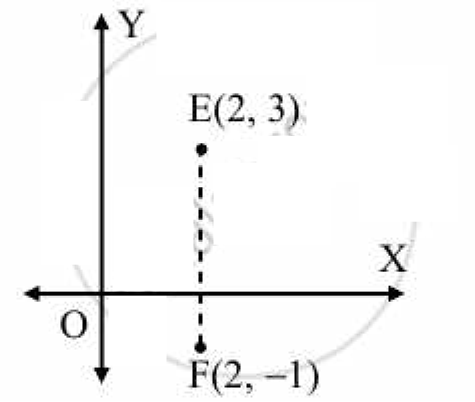
= $\frac{7-3}{5-(-2)}$

= $\frac{4}{7}$.

Exercise 5.2 | Q 1.3 | Page 69
Find the slope of the following lines which pass through the point: (2, 3), (2, – 1)

SOLUTION

Let E = (2, 3) = (x_1, y_1) and F = (2, – 1) = (x_2, y_2)
Since $x_1 = x_2 = 2$
∴ The slope of EF is not defined. ...[EF || y-axis]



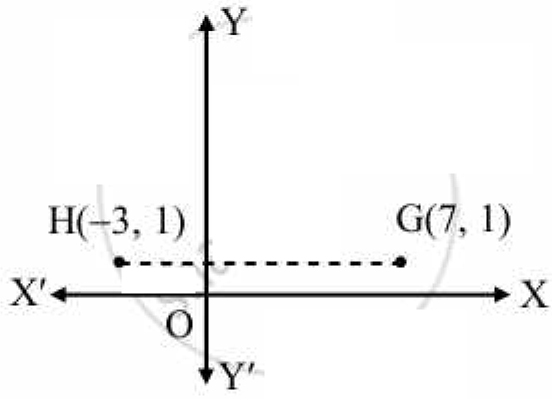
Exercise 5.2 | Q 1.4 | Page 69

Find the slope of the following lines which pass through the point: (7, 1), (– 3, 1)

SOLUTION

Let, G = (7, 1) = (x_1, y_1) and H = (– 3, 1) = (x_2, y_2) say.
Since $y_1 = y_2$

∴ The slope of GH = 0. ...[GH || x-axis]



Exercise 5.2 | Q 2 | Page 69

If the X and Y-intercepts of line L are 2 and 3 respectively, then find the slope of line L.

SOLUTION

Given,
x-intercept of line L is 2 and
y-intercept of line L is 3
∴ the line L intersects X-axis at (2, 0) and Y-axis at (0, 3).
i.e. the line L passes through (2, 0) = (x_1, y_1) and (0, 3) = (x_2, y_2) say.

Slope of line L = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{3-0}{0-2}$

$= \frac{-3}{2}$.

Exercise 5.2 | Q 3 | Page 69

Find the slope of the line whose inclination is 30°.

SOLUTION

Given, inclination (θ) = 30°

Slope of the line = tan θ = tan30° = $\frac{1}{\sqrt{3}}$.

Exercise 5.2 | Q 4 | Page 69

Find the slope of the line whose inclination is 45°.

SOLUTION

Given, inclination (θ) = 45°
Slope of the line = tan θ = tan 45° = 1.

Exercise 5.2 | Q 5 | Page 69

A line makes intercepts 3 and 3 on the co-ordinate axes. Find the slope of the line.

SOLUTION

Given,
x-intercept of line is 3 and
y-intercept of line is 3
∴ The line intersects X-axis at (3, 0) and Y-axis at (0, 3).
i.e. the line passes through (3, 0) = (x_1, y_1) and (0, 3) = (x_2, y_2) say.

Given, inclination (θ) = 30°

Slope of the line = tan θ = tan30° = $\dfrac{1}{\sqrt{3}}$.

Exercise 5.2 | Q 6 | Page 69

Without using Pythagoras theorem, show that points A (4, 4), B (3, 5) and C (– 1, – 1) are the vertices of a right-angled triangle.

SOLUTION

Given, A(4, 4) = (x_1, y_1), B(3, 5) = (x_2, y_2), C (– 1, – 1) = (x_3, y_3)

Slope of AB = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-4}{3-4} = -1$

Slope of BC = $\dfrac{y_3 - y_2}{x_3 - x_2} = \dfrac{-1-5}{-1-3} = \dfrac{-6}{-4} = \dfrac{3}{2}$

Slope of AC = $\dfrac{y_3 - y_1}{x_3 - x_1} = \dfrac{-1-4}{-1-4} = \dfrac{-5}{-5} = 1$

Slope of AB x slope of AC = – 1 x 1 = – 1

∴ side AB ⊥ side AC

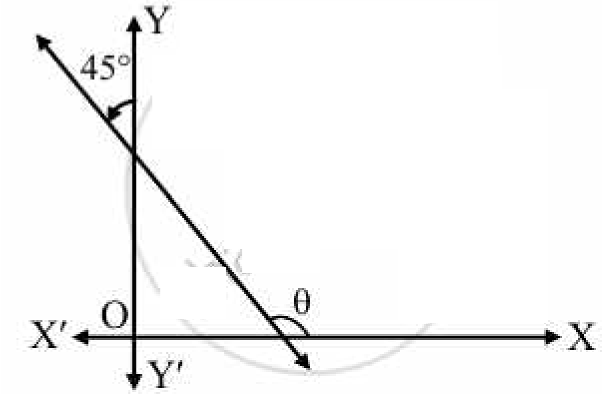
∴ ΔABC is a right angled triangle, right angled at A.

∴ The given points are the vertices of a right angled triangle.

Exercise 5.2 | Q 7 | Page 69

Find the slope of the line which makes angle of 45° with the positive direction of the Y-axis measured clockwise.

SOLUTION



Since, the line makes an angle of 45° with positive direction of Y-axis in anticlockwise direction.

∴ Inclination of the line (θ) = (90° + 45°)

∴ Slope of the line = tan(90° + 45°)

= – cot 45° ...[tan (90 + θ°) = – cot θ]

= – 1.

Exercise 5.2 | Q 8 | Page 70

Find the value of k for which the points P(k, – 1), Q(2, 1) and R(4, 5) are collinear.

SOLUTION

Given, points P(k, – 1), Q(2, 1) and R(4, 5) are collinear.

∴ Slope of PQ = Slope of QR

$$\therefore \frac{1-(-1)}{2-k} = \frac{5-1}{4-2}$$

$$\therefore \frac{2}{2-k} = \frac{4}{2}$$

$\therefore 1 = 2 - k$

$\therefore\ k = 2 - 1 = 1.$

EXERCISE 5.3 [PAGE 73]

Exercise 5.3 | Q 1.1 | Page 73

Write the equation of the line: parallel to the X-axis and at a distance of 5 units from it and above it.

SOLUTION

Equation of a line parallel to X-axis is y = k.
Since, the line is at a distance of 5 units above X-axis.
∴ k = 5
∴ the equation of the required line is y = 5.

Exercise 5.3 | Q 1.2 | Page 73

Write the equation of the line: parallel to the Y-axis and at a distance of 5 units from it and to the left of it.

SOLUTION

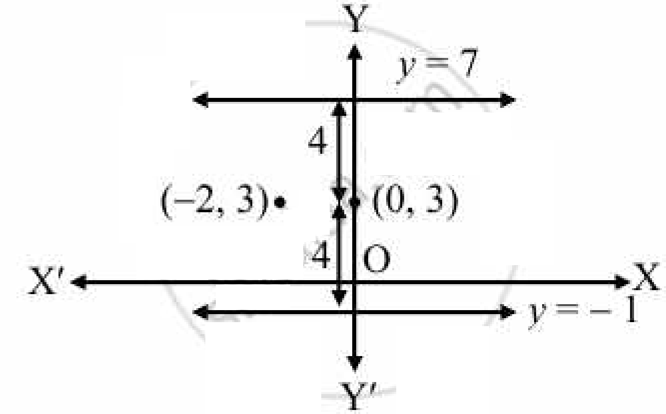
Equation of a line parallel to Y-axis is x = h.
Since, the line is at a distance of 5 units to the left of Y-axis.
∴ h = – 5
∴ the equation of the required line is x = – 5.

Exercise 5.3 | Q 1.3 | Page 73

Write the equation of the line: parallel to the X-axis and at a distance of 4 units from the point (− 2, 3).

SOLUTION

Equation of a line parallel to the X-axis is of the form y = k (k > 0 or k < 0).
Since, the line is at a distance of 4 units from the point (− 2, 3).
∴ k = 3 + 4 = 7 or k = 3 − 4 = − 1
∴ the equation of the required line is y = 7 or y = − 1.



Exercise 5.3 | Q 2.1 | Page 73

Obtain the equation of the line: parallel to the X-axis and making an intercept of 3 units on the Y-axis.

SOLUTION

Equation of a line parallel to X-axis with y-intercept 'k' is y = k.
Here, y-intercept = 3
∴ the equation of the required line is y = 3.

Exercise 5.3 | Q 2.2 | Page 73

Obtain the equation of the line: parallel to the Y-axis and making an intercept of 4 units on the X-axis.

SOLUTION

Equation of a line parallel to Y-axis with x-intercept 'h' is x = h.
Here, x-intercept = 4
∴ the equation of the required line is x = 4.

Exercise 5.3 | Q 3.1 | Page 73
Obtain the equation of the line containing the point: A(2, – 3) and parallel to the Y-axis.

SOLUTION

Equation of a line parallel to Y-axis is of the form x = h.
Since, the line passes through A(2, – 3).
∴ h = 2
∴ the equation of the required line is x = 2.

Exercise 5.3 | Q 3.2 | Page 73
Obtain the equation of the line containing the point: B(4, – 3) and parallel to the X-axis.

SOLUTION

Equation of a line parallel to X-axis is of the form y = k.
Since, the line passes through B(4, – 3)
∴ k = – 3
∴ the equation of the required line is y = – 3.

Exercise 5.3 | Q 4 | Page 73
Find the equation of the line passing through the points A(2, 0) and B(3, 4).

SOLUTION

The required line passes through the points A(2, 0) = (x_1, y_1) and B(3, 4) = (x_2, y_2) say.
Equation of the line in two point form is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

∴ the equation of the required line is

$$\frac{y-0}{4-0}=\frac{x-2}{3-2}$$

$$\therefore \frac{y}{4}=\frac{x-2}{1}$$

∴ y = 4(x – 2)

∴ y = 4x – 8

∴ 4x – y – 8 = 0.

Exercise 5.3 | Q 5 | Page 73

Line y = mx + c passes through the points A(2, 1) and B(3, 2). Determine m and c.

SOLUTION

Given, A(2, 1) and B(3, 2).

Equation of a line in two point form is

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

∴ the equation of the passing through A and B line is

$$\frac{y-1}{2-1}=\frac{x-2}{3-2}$$

$$\therefore \frac{y-1}{1}=\frac{x-2}{1}$$

∴ y – 1 = x – 2

∴ y = x – 1

Comparing this equation with y = mx + c, we get

m = 1 and c = – 1

Alternative method:

Points A(2, 1) and B(3, 2) lie on the line y = mx + c.

∴ They must satisfy the equation.

$\therefore$ 2m + c = 1 ...(i)

and 3m + c = 2 ...(ii)

equation (ii) equation (i) gives m = 1

Substituting m = 1 in (i), we get

2(1) + c = 1

$\therefore$ c = 1 – 2 = – 1.

Exercise 5.3 | Q 6.1 | Page 73

The vertices of a triangle are A(3, 4), B(2, 0) and C(–1, 6). Find the equations of side BC

SOLUTION

Vertices of ΔABC are A(3, 4), B(2, 0) and C(–1, 6).
Equation of a line in two point form is

$$\frac{y-y_1}{y_2-y_1}=\frac{x-1}{x_2-x_1}$$

$\therefore$ the equation of the side BC is

$$\frac{y-0}{6-0}=\frac{x-2}{-1-2}\quad\ldots\begin{bmatrix}\mathrm{B}=(x_1,y_1)=(2,0)\\\mathrm{C}=(x_2,y_2)=(-1,6)\end{bmatrix}$$

$$\therefore\ \frac{y}{6}=\frac{x-2}{-3}$$

y = – 2 (x – 2)

$\therefore$ 2x + y – 4 = 0.

Exercise 5.3 | Q 6.2 | Page 73

The vertices of a triangle are A(3, 4), B(2, 0) and C(–1, 6). Find the equations of the median AD.

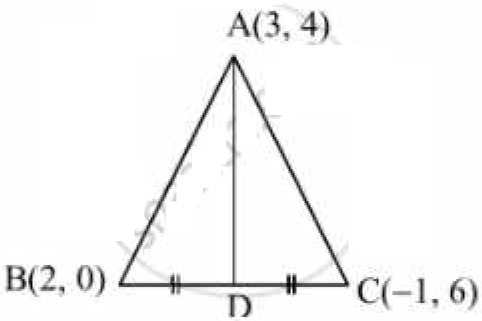
SOLUTION

Vertices of ΔABC are A(3, 4), B(2, 0) and C(–1, 6).
Let D be the midpoint of side BC.
Then, AD is the median through A.

$\therefore$ D = $\left(\frac{2-1}{2}, \frac{0+6}{2}\right) = \left(\frac{1}{2}, 3\right)$

The median AD passes through the points

A(3, 4) and $D\left(\frac{1}{2}, 3\right)$.



$\therefore$ the equation of the median AD is

$$\frac{y-4}{3-4} = \frac{x-3}{\frac{1}{2}-3}$$

$$\therefore \frac{y-4}{-1} = \frac{x-3}{-\frac{5}{2}}$$

$$\therefore \frac{5}{2}(y-4)\ x - 3$$

$\therefore$ 5y – 20 = 2x – 6

$\therefore$ 2x – 5y + 14 = 0.

Exercise 5.3 | Q 6.3 | Page 73

The vertices of a triangle are A(3, 4), B(2, 0) and C(–1, 6). Find the equations of the midpoints of sides AB and BC.

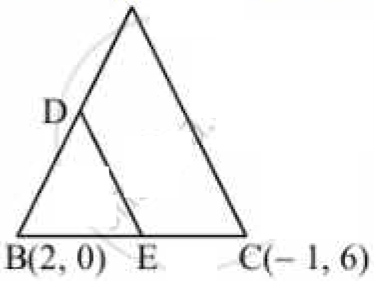
SOLUTION

Vertices of ΔABC are A(3, 4), B(2, 0) and C(–1, 6).
Let D and E be the midpoints of side AB and side BC respectively.

$\therefore$ D = $\left(\frac{3+2}{2}, \frac{4+0}{2}\right) = \left(\frac{5}{2}, 2\right)$ and

E = $\left(\frac{2-1}{2}, \frac{0+6}{2}\right) = \left(\frac{1}{2}, 3\right)$

∴ the equation of the line DE is A(3, 4)



$$\frac{y-2}{3-2} = \frac{x-\frac{5}{2}}{\frac{1}{2}-\frac{5}{2}}$$

$$\therefore \frac{y-2}{1} = \frac{2x-5}{-4}$$

∴ – 4(y – 2) = 2x – 5

∴ – 4y + 8 = 2x – 5

∴ 2x + 4y – 13 = 0.

Exercise 5.3 | Q 7.1 | Page 73

Find the x and y-intercepts of the following line: $\frac{x}{y} + \frac{y}{2}$ = 1

SOLUTION

Given equation of the line is $\frac{x}{y} + \frac{y}{2}$ = 1

This is of the form $\frac{x}{\mathrm{a}} + \frac{y}{\mathrm{b}}$ = 1,

where x-intercept = a, y-intercept = b

∴ x-intercept = 3, y-intercept = 2.

Exercise 5.3 | Q 7.2 | Page 73

Find the x and y-intercepts of the following line: $\frac{3x}{2} + \frac{2y}{3}$ = 1

SOLUTION

Given equation of the line is $\frac{3x}{2} + \frac{2y}{3} = 1$

$\therefore \frac{x}{\left(\frac{2}{3}\right)} + \frac{y}{\left(\frac{3}{2}\right)} = 1$

This is of the form $\frac{x}{a} + \frac{y}{b} = 1$,

where x-intercept = a, y-intercept = b

$\therefore$ x-intercept = $\frac{2}{3}$ and y-intercept = $\frac{3}{2}$.

Exercise 5.3 | Q 7.3 | Page 73

Find the x and y-intercepts of the following line: $2x - 3y + 12 = 0$

SOLUTION

Given equation of the line is $2x - 3y + 12 = 0$

$\therefore 2x - 3y = -12$

$\therefore \frac{2x}{(-12)} - \frac{3y}{(-12)} = 1$

$\therefore \frac{x}{-6} + \frac{y}{4} = 1$

This is of the form $\frac{x}{a} + \frac{y}{b} = 1$,

where x-intercept = a, y-intercept = b

$\therefore$ x-intercept = □ 6 and y-intercept = 4.

Exercise 5.3 | Q 8 | Page 73

Find the equations of a line containing the point A(3, 4) and making equal intercepts on the co-ordinate axes.

SOLUTION

Let the equation of the line be

$\frac{x}{a} + \frac{y}{b} = 1$

Since, the required line make equal intercepts on the co-ordinate axes.

$\therefore$ a = b

$\therefore$ (i) reduces to x + y = a ...(ii)

Since the line passes through A(3, 4).

$\therefore$ 3 + 4 = a

i.e. a = 7

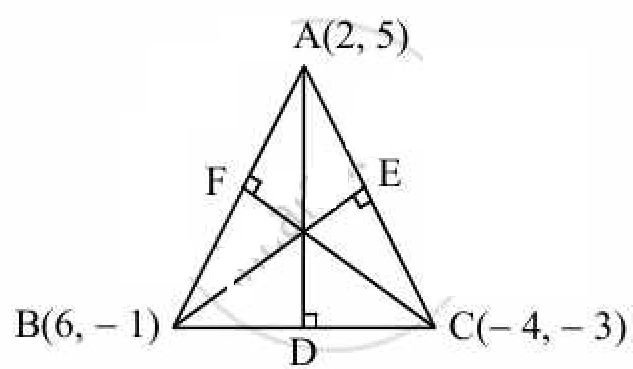
Substituting a = 7 in (ii) to get

x + y = 7.

Exercise 5.3 | Q 9 | Page 73

Find the equations of the altitudes of the triangle whose vertices are A(2, 5), B(6, – 1) and C(– 4, – 3).

SOLUTION



A(2, 5), B(6, – 1), C(– 4, – 3) are the vertices of ΔABC.

Let AD, BE and CF be the altitudes through the vertices A, B and C respectively of ΔABC.

Slope of BC = $\frac{-3-(-1)}{4-6} = \frac{-2}{-10} = \frac{1}{5}$

$\therefore$ slope of AD = – 5 ....[$\because$ AD $\perp$ BC]

Since, altitude AD passes through (2, 5) and has slope – 5.

$\therefore$ the equation of the altitude AD is y – 5 = – 5 (x – 2)

∴ y – 5 = – 5x + 10

∴ 5x + y – 15 = 0

Now, slope of AC = $\frac{-3-5}{-4-2} = \frac{-8}{-6} = \frac{4}{3}$

∴ Slope of BE = $\frac{-3}{4}$ ....[∵ BE ⊥ AC]

Since, altitude BE passes through (6, – 1) and has slope $\frac{-3}{4}$.

∴ the equation of the altitude BE is

y – (– 1) = $\frac{-3}{4}(x-6)$

∴ 4(y + 1) = – 3 (x – 6)

∴ 3x + 4y – 14 = 0

Also, slope of AB = $\frac{-1-5}{6-2} = \frac{-6}{4} = \frac{-3}{2}$

∴ Slope of BE = $\frac{2}{3}$ ....[∵ CF ⊥ AB]

Since, altitude CF passes through (– 4, – 3) and has slope $\frac{2}{3}$.

∴ the equation of the altitude CF is

y – (– 3) = $\frac{2}{3}[x-(-4)]$

∴ 3 (y + 3) = 2 (x + 4)

∴ 2x – 3y – 1 = 0.

EXERCISE 5.4 [PAGE 78]

Exercise 5.4 | Q 1.1 | Page 78

Find the slope, x-intercept, y-intercept of the following line : 2x + 3y – 6 = 0

SOLUTION

Given equation of the line is 2x + 3y – 6 = 0

Comparing this equation with ax + by + c = 0, we get

a = 2, b = 3, c = – 6

$\therefore$ Slope of the line = $\frac{-a}{b} = \frac{-2}{3}$

x-intercept = $\frac{-c}{a} = \frac{-(-6)}{2}$ = 3

y-intercept = $\frac{-c}{b} = \frac{-(-6)}{3}$ = 2

Exercise 5.4 | Q 1.2 | Page 78

Find the slope, x-intercept, y-intercept of the following line : x + 2y = 0

SOLUTION

Given equation of the line is x + 2y = 0

Comparing this equation with ax + by + c = 0,

we get

a = 1, b = 2, c = 0

$\therefore$ Slope of the line = $\frac{-a}{b} = \frac{-1}{2}$

x-intercept = $\frac{-c}{a} = \frac{0}{1}$ = 0

y-intercept = $\frac{-c}{b} = \frac{0}{2}$ = 0

Exercise 5.4 | Q 2.1 | Page 78

Write the following equation in ax + by + c = 0 form: y = 2x – 4

SOLUTION

y = 2x – 4

$\therefore$ 2x – y – 4 = 0 is the equation in ax + by + c = 0 form.

Exercise 5.4 | Q 2.2 | Page 78

Write the following equation in ax + by + c = 0 form: y = 4

SOLUTION

y = 4
∴ 0x +1y – 4 = 0 is the equation in ax + by + c = 0 form.

Exercise 5.4 | Q 2.3 | Page 78

Write the following equation in ax + by + c = 0 form: $\frac{x}{2} + \frac{y}{4} = 1$

SOLUTION

$\frac{x}{2} + \frac{y}{4} = 1$

$\therefore \frac{2x + y}{4} = 1$

∴ 2x + y = 4

∴ 2x + y – 4 = 0 is the equation in ax + by + c = 0 form.

Exercise 5.4 | Q 2.4 | Page 78

Write the following equation in ax + by + c = 0 form: $\frac{x}{3} = \frac{y}{2}$

SOLUTION

$\frac{x}{3} = \frac{y}{2}$

∴ 2x = 3y

∴ 2x – 3y + 0 = 0 is the equation in ax + by + c = 0 form.

Exercise 5.4 | Q 3 | Page 78

Show that the lines x – 2y – 7 = 0 and 2x − 4y + 5 = 0 are parallel to each other.

SOLUTION

Let m_1 be the slope of the line x – 2y – 7 = 0.

$\therefore m_1 = \frac{-2}{-2} = \frac{1}{2}$

Let m_2 be the slope of the line 2x – 4y + 5 = 0.

$\therefore$ $m_2 = \frac{-2}{-4} = \frac{1}{2}$

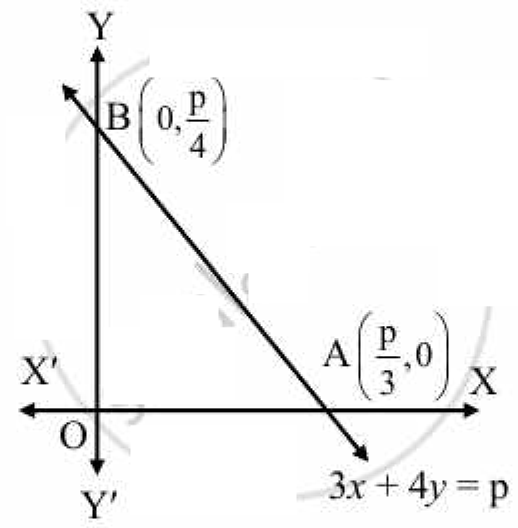
Since, $m_1 = m_2$

$\therefore$ The given lines are parallel to each other.

Exercise 5.4 | Q 4 | Page 78

If the line 3x + 4y = p makes a triangle of area 24 square units with the co-ordinate axes, then find the value of p.

SOLUTION



Let the line 3x + 4y = p cuts the X and Y-axes at points A and B respectively.
3x + 4y = p

$\therefore \frac{3x}{p} + \frac{4y}{p} = 1$

$\therefore \frac{x}{\frac{p}{3}} + \frac{y}{\frac{p}{4}} = 1$

This equation is of the form

$\frac{x}{a} + \frac{y}{b} = 1.$

with $a = \frac{p}{3}$ and $b = \frac{p}{4}$

$\therefore$ A = $(a, 0) = \left(\frac{p}{3}, 0\right)$ and $B = (0, b) = \left(0, \frac{p}{4}\right)$

$\therefore$ OA = $\frac{p}{4}$ and $OB = \frac{p}{4}$

Given, A (Δ.OAB) = 24 sq. units

$\therefore \frac{1}{2} \times OA \times OB = 24$

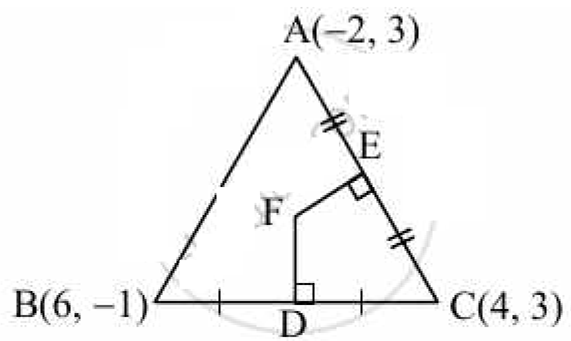
$\therefore \frac{1}{2} \times \frac{p}{3} \times \frac{p}{4} = 24$

$\therefore p^2 = 576$

$\therefore p = \pm 24.$

Exercise 5.4 | Q 5 | Page 78
Find the co-ordinates of the circumcentre of the triangle whose vertices are A(– 2, 3), B(6, – 1), C(4, 3).

SOLUTION



Here, A(– 2, 3), B(6, – 1), C(4, 3) are the verticals of ΔABC.
Let F be the circumcentre of ΔABC.

Let FD and FE be the perpendicular bisectors of the sides BC and AC respectively.
∴ D and E are the midpoints of side BC and AC respectively.

$\therefore D = \left(\frac{6+4}{2}, \frac{-1+3}{2}\right)$

$\therefore D = (5, 1)$ and $E = \left(\frac{-2+4}{2}, \frac{3+3}{2}\right)$

$\therefore$ E = (1, 3)

Now, slope of BC = $\frac{-1-3}{6-4}$ = – 2

$\therefore$ slope of FD = $\frac{1}{2}$ ...[$\because$ FD $\perp$ BC]

Since, FD passes through (5, 1) and has slope $\frac{1}{2}$

$\therefore$ Equation of FD is y – 1 = $\frac{1}{2}(x-5)$

$\therefore$ 2(y – 1) = x – 5

$\therefore$ x – 2y – 3 = 0 ....(i)

Since, both the points A and C have same y co-ordinates i.e. 3

$\therefore$ the points A and C lie on the line y = 3.

Since, FE passes through E(1, 3).

$\therefore$ the equation of FE is x = 1. ...(ii)

To find co-ordinates of circumcentre, we have to solve equations (i) and (ii).

Substituting the value of x in (i), we get

1 – 2y – 3 = 0

$\therefore$ y = – 1

$\therefore$ Co-ordinates of circumcentre F ≡ (1, – 1).

Exercise 5.4 | Q 6 | Page 78

Find the equation of the line whose x-intercept is 3 and which is perpendicular to the line 3x – y + 23 = 0.

SOLUTION

Slope of the line 3x – y + 23 = 0 is 3.

$\therefore$ slope of the required line which is perpendicular to 3x – y + 23 = 0 is -1/3

Since, the x-intercept of the required line is 3.

$\therefore$ it passes through (3, 0).

$\therefore$ the equation of the required line is

$$y - 0 = \frac{-1}{3}(x - 3)$$

$\therefore 3y = -x + 3$

$\therefore x + 3y = 3.$

Exercise 5.4 | Q 7 | Page 78

Find the distance of the point A(– 2, 3) from the line 12x – 5y – 13 = 0.

SOLUTION

Let p be the perpendicular distance of the point A(– 2, 3) from the line 12x – 5y – 13 = 0

Here, a = 12, b = – 5, c = – 13, x_1 = – 2, y_1 = 3

$$\therefore p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{12(-2) - 5(3) - 13}{\sqrt{12^2 + (-5)^2}}\right|$$

$$= \left|\frac{-24 - 15 - 13}{\sqrt{144 + 25}}\right|$$

$$= \left|\frac{-52}{13}\right|$$

= 4 units.

Exercise 5.4 | Q 8 | Page 78

Find the distance between parallel lines 9x + 6y − 7 = 0 and 9x + 6y − 32 = 0.

SOLUTION

Equations of the given parallel lines are 9x + 6y − 7 = 0 and 9x + 6y − 32 = 0.

Here, a = 9, b = 6, C_1 = –7 and C_2 = –32

$\therefore$ Distance between the parallel lines

$$= \left| \frac{C_1 - C_2}{\sqrt{a^2 + b^2}} \right|$$
$$= \left| \frac{-7 - (-32)}{\sqrt{9^2 + 6^2}} \right|$$
$$= \left| \frac{-7 + 32}{\sqrt{81 + 36}} \right|$$
$$= \left| \frac{25}{\sqrt{117}} \right|$$
$$= \frac{25}{\sqrt{117}} \text{ units.}$$

Exercise 5.4 | Q 9 | Page 78

Find the equation of the line passing through the point of intersection of lines x + y – 2 = 0 and 2x – 3y + 4 = 0 and making intercept 3 on the X-axis.

SOLUTION

Given equations of lines are

$x + y - 2 = 0$ ...(i)
and $2x - 3y + 4 = 0$ ...(ii)

Multiplying equation (i) by 3, we get
$3x + 3y - 6 = 0$ ...(iii)

Adding equation (ii) and (iii), we get
$5x - 2 = 0$

$$\therefore x = \frac{2}{5}$$

Substituting $x = \frac{2}{5}$ in equation (i), we get

$$\frac{2}{5} + y - 2 = 0$$

$\therefore$ y = $2 - \frac{2}{5} = \frac{8}{5}$

$\therefore$ The required line passes through point $\left(\frac{2}{5}, \frac{8}{5}\right)$.

Also, the line makes intercept of 3 on X-axis

$\therefore$ it also passes through point (3, 0).

$\therefore$ required equation of line passing through points $\left(\frac{2}{5}, \frac{8}{5}\right)$ and (3, 0) is

$$\frac{y - \frac{8}{5}}{0 - \frac{8}{5}} = \frac{x - \frac{2}{5}}{3 - \frac{2}{5}}$$

$$\therefore \frac{\frac{5y-8}{5}}{-\frac{8}{5}} = \frac{\frac{5x-2}{5}}{\frac{13}{5}}$$

$$\therefore \frac{5y-8}{-8} = \frac{5x-2}{13}$$

$\therefore$ 13 (5y – 8) = – 8 (5x – 2)

$\therefore$ 65y – 104 = – 40x + 16

$\therefore$ 40x + 65y – 120 = 0

$\therefore$ 8x + 13y – 24 = 0 which is the equation of the required line.

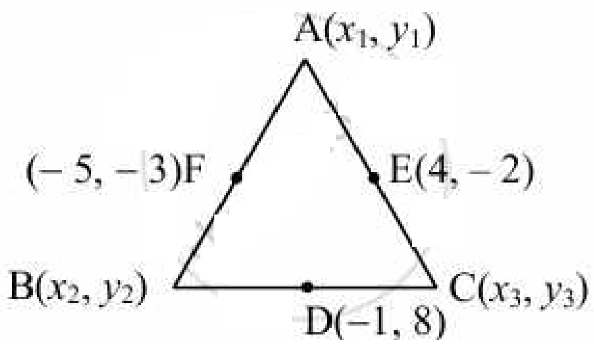
Exercise 5.4 | Q 10.1 | Page 78

D(– 1, 8), E(4, – 2), F(– 5, – 3) are midpoints of sides BC, CA and AB of ΔABC
Find: equations of sides of ΔABC

SOLUTION

Let A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the vertices of ΔABC.

Given, points D, E and F are midpoints of sides BC, CA and AB respectively of ΔABC.



$$D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

$$\therefore (-1, 8) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

$\therefore x_2 + x_3 = -2$ ...(i)

and $y_2 + y_3 = 16$ ...(ii)

$$\text{Also, } E = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$

$$\therefore (4, -2) = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$$

$\therefore x1 + x3 = 8$ ...(iii)

and $y1 + y3 = -4$ ...(iv)

$$\text{Similarly, } F = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\therefore (-5, -3) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$\therefore x_1 + x_2 = -10$ ...(v)

and $y_1 + y_2 = -6$ ...(vi)

For x-coordinates:

Adding (i), (iii) and (v), we get

$2x_1 + 2x_2 + 2x_3 = -4$

$\therefore x_1 + x_2 + x_3 = -2$

Solving (i) and (vii), we get

$x_1 = 0$

Solving (iii) and (vii), we get
$x_2 = -10$
Solving (v) and (vii), we get
$x_3 = 8$

For y-coordinates:
Adding (ii), (iv) and (vi), we get

$2y_1 + 2y_2 + 2y_3 = 6$
$\therefore y_1 + y_2 + y_3 = 3$ ...(viii)
Solving (ii) and (viii), we get
$y_1 = -13$
Solving (iv) and (viii), we get
$y_2 = 7$
Solving (vi) and (viii), we get
$y_3 = 9$
∴ Vertices of ΔABC are A.(0, – 13), B(– 10, 7), C(8, 9)
a. Equation of side AB is

$$\frac{y+13}{7+13} = \frac{x-0}{-10-0}$$

$$\therefore \frac{y+13}{20} = \frac{x}{-10}$$

$$\therefore \frac{y+13}{2} = -x$$

$\therefore 2x + y + 13 = 0$

b. Equation of side BC is

$$\frac{y-7}{9-7} = \frac{x+10}{8+10}$$

$$\therefore \frac{y-7}{2} = \frac{x+10}{9}$$

$$\therefore y-7 = \frac{x+10}{9}$$

$\therefore x - 9y + 73 = 0$

c. Equation of side AC is

$$\frac{y+13}{9+13} = \frac{x-0}{8-0}$$

$\therefore \quad \dfrac{y+13}{22} = \dfrac{x}{8}$

∴ 8(y + 130 = 22x

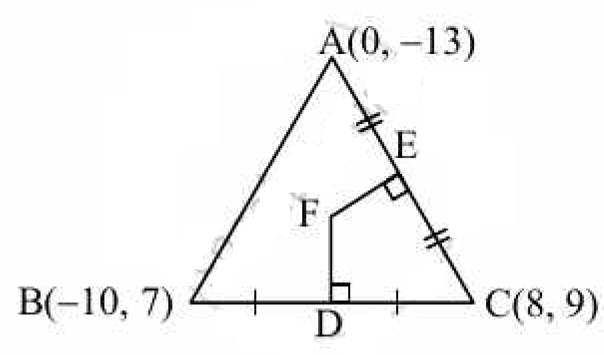
∴ 4(y + 13) = 11x

∴ 11x – 4y – 52 = 0.

Exercise 5.4 | Q 10.2 | Page 78

D(– 1, 8), E(4, – 2), F(– 5, – 3) are midpoints of sides BC, CA and AB of ΔABC Find: co-ordinates of the circumcentre of ΔABC.

SOLUTION



Here, A(0, – 13) B(– 10, 7), C(8, 9) are the vertices of ΔABC.
Let F be the circumcentre of ΔABC.

Let FD and FE be perpendicular bisectors of the sides BC and AC respectively.
∴ D and E are the midpoints of side BC and AC.

$\therefore$ D = $\left(\dfrac{-10+8}{2}, \dfrac{7+9}{2}\right)$

$\therefore$ D = (– 1 , 8) and E = $\left(\dfrac{0+8}{2}, \dfrac{-13+9}{2}\right)$

∴ E = (4, – 2)

Now, slope of BC = $\dfrac{7-9}{-10-8} = \dfrac{1}{9}$

∴ slope of FD = – 9 ...[∵ FD ⊥ BC]

Since, FD passes through (– 1, 8) and has slope – 9

∴ Equation of FD is

y – 8 = – 9 (x + 1)

∴ y – 8 = – 9 x – 9

∴ y = – 9x – 1 ...(i)

Also, slope of AC = $\frac{-13-9}{0-8} = \frac{11}{4}$

∴ Slope of FE = $\frac{-4}{11}$ ...[∵ FE ⊥ AC]

Since, FE passes through (4, – 2) and has slope $\frac{-4}{11}$

∴ Equation of FE is

y + 2 = $\frac{-4}{11}(x-4)$

∴ 11(y + 2) = – 4 (x – 4)

∴ 11y + 22 = – 4x + 16

∴ 4x + 11y = – 6 ...(ii)

To find co-ordinates of circumcentre,

we have to solve equations (i) and (ii).

Substituting the value of y in (ii), we get

4x + 11(– 9x – 1) = – 6

∴ 4x – 99x – 11 = – 6

∴ –95x = 5

∴ x = $\frac{-1}{19}$

Substituting the value of x in (i), we get

y = $-9\left(-\frac{1}{19}\right) - 1 = \frac{-10}{19}$

∴ Co-ordinates of circumcentre F ≡ $\left(\frac{-1}{19}, \frac{-10}{19}\right)$.

MISCELLANEOUS EXERCISE 5 [PAGES 79 - 80]

Miscellaneous Exercise 5 | Q 1.1 | Page 79

Find the slope of the line passing through the following point: (1, 2), (3, – 5)

SOLUTION

Let A = (1, 2) = (x_1, y_1) and B = (3, – 5) = (x_2, y_2) say.

Slope of line AB = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-5-2}{3-1}$

$= \frac{-7}{2}$.

Miscellaneous Exercise 5 | Q 1.2 | Page 79

Find the slope of the line passing through the following point: (1, 3), (5, 2)

SOLUTION

Let C = (1, 3) = (x_1, y_1) and D = (5, 2) = (x_2, y_2) say.

Slope of line CD = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-3}{5-1}$

$= \frac{-1}{4}$.

Miscellaneous Exercise 5 | Q 1.3 | Page 79

Find the slope of the line passing through the following point: (–1, 3), (3, –1)

SOLUTION

Let E =(–1, 3) = (x_1, y_1) and F = (3, –1) = (x_2, y_2) say.

Slope of line EF = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-3}{3-(-1)}$

$= \frac{-4}{4}$

$= -1.$

Miscellaneous Exercise 5 | Q 1.4 | Page 79
Find the slope of the line passing through the following point: (2, – 5), (3, – 1)

SOLUTION

Let P = (2, – 5) = (x_1, y_1) and Q = (3, – 1) = (x_2, y_2) say.

Slope of line PQ $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-1-(-5)}{3-2}$

$= \frac{-1+5}{1}$

$= 4.$

Miscellaneous Exercise 5 | Q 2.1 | Page 79
Find the slope of the line which makes an angle of 120° with the positive X-axis.

SOLUTION

θ = 120°
Slope of the line = tan 120°
= tan (180 – 60°)
= – tan 60° ...[tan(180° – θ) = – tan θ]
= – √3.

Miscellaneous Exercise 5 | Q 2.2 | Page 79
Find the slope of the line which makes intercepts 3 and – 4 on the axes.

SOLUTION

Given, x-intercept of line is 3
and y-intercept of line is – 4
∴ The line intersects X-axis at (3, 0) and Y-axis at (0, – 4).
∴ The line passes through (3, 0) = (x_1, y_1) and (0, – 4) = (x_2, y_2) say.

$\therefore$ Slope of line = $\dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-4 - 0}{0 - 3}$

$= \dfrac{-4}{-3}$

$= \dfrac{4}{3}$.

Miscellaneous Exercise 5 | Q 2.3 | Page 79

Find the slope of the line which passes through the points A(–2, 1) and the origin.

SOLUTION

Required line passes through O(0, 0) = (x_1, y_1) and A(– 2, 1) = (x_2, y_1) say.

Slope of line OA = $\dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{1 - 0}{-2 - 0}$

$= \dfrac{1}{-2}$

$= \dfrac{-1}{2}$.

Miscellaneous Exercise 5 | Q 3.1 | Page 79

Find the value of k: if the slope of the line passing through the points (3, 4), (5, k) is 9.

SOLUTION

Let P(3, 4), Q(5, k).
Slope of PQ = 9 …[Given]

$\therefore \dfrac{k - 4}{5 - 3} = 9$

$\therefore \dfrac{k - 4}{2} = 9$

$\therefore$ k – 4 = 18

$\therefore$ k = 22.

Miscellaneous Exercise 5 | Q 3.2 | Page 79

Find the value of k: the points (1, 3), (4, 1), (3, k) are collinear.

SOLUTION

The points A(1, 3), B(4, 1) and C(3, k) are collinear.

∴ Slope of AB = Slope of BC

$$\therefore \frac{1-3}{4-1} = \frac{k-1}{3-4}$$

$$\therefore \frac{-2}{3} = \frac{k-1}{-1}$$

$$\therefore 2 = 3k - 3$$

$$\therefore k = \frac{5}{3}.$$

Miscellaneous Exercise 5 | Q 3.3 | Page 79

Find the value of k: the point P(1, k) lies on the line passing through the points A(2, 2) and B(3, 3).

SOLUTION

Given, point P(1, k) lies on the line joining A(2, 2) and B(3, 3).

∴ Slope of AB = Slope of BP

$$\therefore \frac{3-2}{3-2} = \frac{3-k}{3-1}$$

$$\therefore 1 = \frac{3-k}{2}$$

$$\therefore 2 = 3 - k$$

$$\therefore k = 1.$$

Miscellaneous Exercise 5 | Q 4 | Page 79

Reduce the equation 6x + 3y + 8 = 0 into slope-intercept form. Hence, find its slope.

SOLUTION

Given equation is 6x + 3y + 8 = 0, which can be written as

3y = – 6x – 8

$\therefore$ y = $\dfrac{-6x}{3} - \dfrac{8}{3}$

$\therefore$ y = $-2x - \dfrac{8}{3}$

This is of the form y = mx + c with m = – 2

$\therefore$ y = $-2x - \dfrac{8}{3}$ is in slope-intercept form with slope = – 2.

Miscellaneous Exercise 5 | Q 5 | Page 79
Verify that A(2, 7) is not a point on the line x + 2y + 2 = 0.

SOLUTION

Given equation is x + 2y + 2 = 0.
Substituting x = 2 and y = 7 in L.H.S. of given equation, we get
L.H.S. = x + 2y + 2
= 2 + 2(7) + 2
= 2 + 14 + 2
= 18
≠ R.H.S.
∴ Point A does not lie on the given line.

Miscellaneous Exercise 5 | Q 6 | Page 79
Find the X-intercept of the line x + 2y – 1 = 0

SOLUTION

Given equation of the line is x + 2y – 1 = 0
To find the x-intercept, put y = 0 in given equation of the line
∴ x + 2(0) – 1 = 0
∴ x + 0 – 1 = 0
∴ x = 1
∴ X-intercept of the given line is 1.
Alternative method:
Given equation of the line is
x + 2y – 1 = 0
i.e. x + 2y = 1

$\therefore \dfrac{x}{1} + \dfrac{y}{\frac{1}{2}}$ = 1

Comparing with $\dfrac{x}{a} + \dfrac{y}{b}$ = 1, we get a = 1

∴ X-intercept of the line is 1.

Miscellaneous Exercise 5 | Q 7 | Page 79
Find the slope of the line y – x + 3 = 0.

SOLUTION

Equation of given line is y – x + 3 = 0
i.e. y = x – 3
Comparing with y = mx + c, we get
m = Slope =1.

Miscellaneous Exercise 5 | Q 8 | Page 79
Does point A(2, 3) lie on the line 3x + 2y – 6 = 0? Give reason.

SOLUTION

Given equation is 3x + 2y – 6 = 0.
Substituting x = 2 and y = 3 in L.H.S. of given equation, we get
L.H.S. = 3x + 2y – 6
= 3(2) + 2(3) – 6
= 6
≠ R.H.S.
∴ Point A does not lie on the given line.

Miscellaneous Exercise 5 | Q 9 | Page 79
Which of the following lines passes through the origin?

1. x = 2
2. y = 3
3. y = x + 2
4. 2x – y = 0

SOLUTION

Any line passing through origin is of the form y = mx or ax + by = 0.
Here in the given option, **2x – y = 0** is in the form ax + by = 0.

Miscellaneous Exercise 5 | Q 10.1 | Page 79
Obtain the equation of the line which is: parallel to the X-axis and 3 units below it.

SOLUTION

Equation of a line parallel to X-axis is y = k.
Since, the line is at a distance of 3 units below X-axis.
∴ k = – 3
∴ the equation of the required line is y = – 3 i.e., y + 3 = 0.

Miscellaneous Exercise 5 | Q 10.2 | Page 79
Obtain the equation of the line which is: Obtain the equation of the line which is:

SOLUTION

Equation of a line parallel to Y-axis is $x = h$.
Since, the line is at a distance of 2 units to the left of Y-axis.
$\therefore h = -2$
$\therefore$ the equation of the required line is $x = -2$ i.e., $x + 2 = 0$.

Miscellaneous Exercise 5 | Q 10.3 | Page 79
Obtain the equation of the line which is: parallel to the X-axis and making an intercept of 5 on the Y-axis.

SOLUTION

Equation of a line parallel to X-axis with y-intercept 'k' is $y = k$.
Here, y-intercept = 5
$\therefore$ the equation of the required line is $y = 5$.

Miscellaneous Exercise 5 | Q 10.4 | Page 79
Obtain the equation of the line which is: parallel to the Y-axis and making an intercept of 3 on the X-axis.

SOLUTION

Equation of a line parallel to Y-axis with x-intercept 'h' is $x = h$.
Here, x-intercept = 3
$\therefore$ the equation of the required line is $x = 3$.

Miscellaneous Exercise 5 | Q 11.1 | Page 79
Obtain the equation of the line containing the point: (2, 3) and parallel to the X−axis.

SOLUTION

Equation of a line parallel to X-axis is of the form $y = k$.
Since, the line passes through (2, 3).
$\therefore k = 3$
$\therefore$ the equation of the required line is $y = 3$.

Miscellaneous Exercise 5 | Q 11.2 | Page 79
Obtain the equation of the line containing the point: (2, 4) and perpendicular to the Y−axis.

SOLUTION

Equation of a line perpendicular to Y-axis i.e., parallel to X-axis, is of the form $y = k$.
Since, the line passes through (2, 4).
$\therefore k = 4$
$\therefore$ the equation of the required line is $y = 4$.

Miscellaneous Exercise 5 | Q 11.3 | Page 79

Obtain the equation of the line containing the point: (2, 5) and perpendicular to the X−axis.

SOLUTION

Equation of a line perpendicular to X-axis
i.e., parallel to Y-axis, is of the form $x = h$.
∴ $h = 2$
∴ the equation of the required line is $x = 2$.

Miscellaneous Exercise 5 | Q 12.1 | Page 79
Find the equation of the line: having slope 5 and containing point A(– 1, 2).

SOLUTION

Given, slope(m) = 5 and the line passes through A(– 1, 2).
Equation of the line in slope point form is $y - y_1 = m(x - x_1)$
∴ the equation of the required line is
$y - 2 = 5(x + 1)$
∴ $y - 2 = 5x + 5$
∴ $5x - y + 7 = 0$.

Miscellaneous Exercise 5 | Q 12.2 | Page 79
Find the equation of the line: containing the point (2, 1) and having slope 13.

SOLUTION

Given, slope(m) = 13 and the line passes through (2, 1).
Equation of the line in slope point form is
$y - y_1 = m(x - x_1)$
∴ the equation of the required line is
$y - 1 = 13(x - 2)$
∴ $y - 1 = 13x - 26$
∴ $13x - y = 25$.

Miscellaneous Exercise 5 | Q 12.3 | Page 79

Find the equation of the line: containing the point T(7, 3) and having inclination 90°.

SOLUTION

Given, Inclination of line = $\theta = 90°$
∴ the required line is parallel to Y-axis (or lies on the Y-axis.)

Equation of a line parallel to Y-axis is of the form $x = h$.
Since, the line passes through (7, 3).

∴ $h = 7$
∴ the equation of the required line is $x = 7$.

Miscellaneous Exercise 5 | Q 12.4 | Page 79
Find the equation of the line: containing the origin and having inclination 90°.

SOLUTION

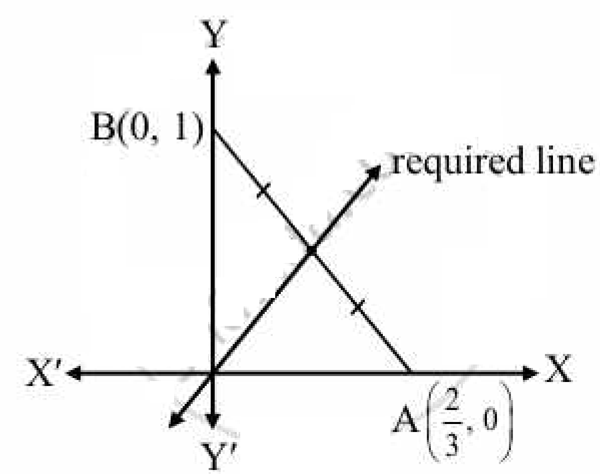
Given, Inclination of line = θ = 90°
∴ the required line is parallel to Y-axis (or lies on the Y-axis.)

Equation of a line parallel to Y-axis is of the form x = h.
Since, the line passes through origin (0, 0).

∴ h = 0
∴ the equation of the required line is x = 0.

Miscellaneous Exercise 5 | Q 12.5 | Page 79
Find the equation of the line: through the origin which bisects the portion of the line 3x + 2y = 2 intercepted between the co-ordinate axes.

SOLUTION



Given equation of the line is 3x + 2y = 2.

$$\therefore \frac{3x}{2} + \frac{2y}{2} = 1$$

$$\therefore \frac{x}{\frac{2}{3}} + \frac{y}{1} = 1$$

This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$, with $a = \frac{2}{3}$, b = 1.

$\therefore$ the line 3x + 2y = 2 intersects the X-axis at $\mathbf{A}\left(\frac{2}{3}, 0\right)$ and Y-axis at B(0, 1).

Required line is passing through the midpoint of AB.

$\therefore$ Midpoint of AB = $\left(\frac{\frac{2}{3}+0}{2}, \frac{0+1}{2}\right) = \left(\frac{1}{3}, \frac{1}{2}\right)$

$\therefore$ Required line passes through (0, 0) and $\left(\frac{1}{3}, \frac{1}{2}\right)$.

Equation of the line in two point form is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x2-x_1}$$

$\therefore$ the equation of the required line is

$$\frac{y-0}{\frac{1}{2}-0} = \frac{x-0}{\frac{1}{3}-0}$$

$\therefore$ 2y = 3x

$\therefore$ 3x –2y = 0.

Miscellaneous Exercise 5 | Q 13 | Page 80

Find the equation of the line passing through the points A(–3, 0) and B(0, 4).

SOLUTION

Since, the required line passes through the points A(–3, 0) and B(0, 4).
Equation of the line in two point form is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

Here, (x_1, y_1) = (–3, 0) and (x_2, y_2) = (0, 4)

$\therefore$ the equation of the required line is

$$\frac{y-0}{4-0} = \frac{x-(-3)}{0-(-3)}$$

$\therefore \dfrac{y}{4} = \dfrac{x+3}{3}$

$\therefore 4x + 12 = 3y$

$\therefore 4x - 3y + 12 = 0.$

Miscellaneous Exercise 5 | Q 14.1 | Page 80

Find the equation of the line: having slope 5 and making intercept 5 on the X−axis.

SOLUTION

Since, the x-intercept of the required line is 5.
∴ it passes through (5, 0).

Also, slope(m) of the line is 5
Equation of the line in slope point form is
$y - y_1 = m(x - x_1)$

∴ the equation of the required line is
$y - 0 = 5(x - 5)$
$\therefore y = 5x - 25$
$\therefore 5x - y - 25 = 0.$

Miscellaneous Exercise 5 | Q 14.2 | Page 80

Find the equation of the line: having an inclination 60° and making intercept 4 on the Y-axis.

SOLUTION

Given, Inclination of line = θ = 60°

∴ slope of the line (m) = tan θ = tan 60° = √3 and the y-intercept of the required line is 4.
∴ it passes through (0, 4).

Equation of the line in slope point form is
$y - y_1 = m(x - x_1)$

∴ the equation of the required line is

$y - 4 = \sqrt{3}(x - 0)$

$\therefore y - 4 = \sqrt{3}x$

$\therefore \sqrt{3}x - y + 4 = 0.$

Miscellaneous Exercise 5 | Q 15.1 | Page 80
The vertices of a triangle are A (1, 4), B (2, 3) and C (1, 6). Find equations of the sides

SOLUTION

Vertices of ΔABC are A(1, 4), B(2, 3) and C(1, 6)

Equation of the line in two point form is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Equation of side AB is

$$\frac{y-4}{3-4} = \frac{x-1}{2-1}$$

$$\therefore \frac{y-4}{-1} = \frac{x-1}{1}$$

$\therefore$ y – 4 = – 1(x – 1)

$\therefore$ x + y = 5

Equation of side BC is

$$\frac{y-3}{6-3} = \frac{x-2}{1-2}$$

$$\therefore \frac{y-3}{3} = \frac{x-2}{-1}$$

$\therefore$ – 1(y – 3) = 3(x – 2)

$\therefore$ 3x + y = 9

Since, both the points A and C have same x co-ordinates i.e. 1

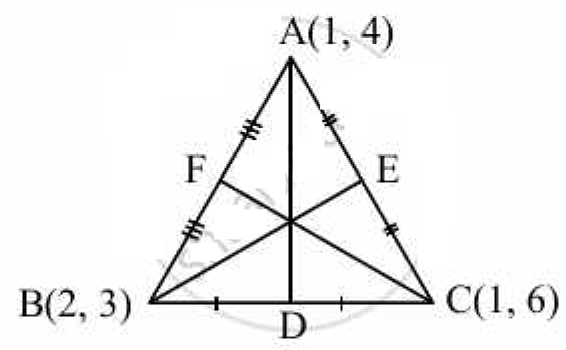
$\therefore$ the points A and C lie on a line parallel to Y-axis.

$\therefore$ the equation of side AC is x = 1.

Miscellaneous Exercise 5 | Q 15.2 | Page 80
The vertices of a triangle are A (1, 4), B (2, 3) and C (1, 6). Find equations of the medians

SOLUTION

Vertices of ΔABC are A(1, 4), B(2, 3) and C(1, 6)
Let D, E and F be the midpoints of sides BC, AC and AB respectively of ΔABC.



Then D = $\left(\frac{2+1}{2}, \frac{3+6}{2}\right) = \left(\frac{3}{2}, \frac{9}{2}\right)$

E = $\left(\frac{1+1}{2}, \frac{6+4}{2}\right)$ = (1 , 5)

F = $\left(\frac{1+2}{2}, \frac{4+3}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

Equation of median AD is

$$\frac{y-4}{\frac{9}{2}-4} = \frac{x-1}{\frac{3}{2}-1}$$

$$\therefore \frac{y-4}{\frac{1}{2}} = \frac{x-1}{\frac{1}{2}}$$

$\therefore$ y − 4 = x − 1

$\therefore$ x −y + 3 = 0

Equation of median BE is

$$\frac{y-3}{5-3} = \frac{x-2}{1-2}$$

$$\therefore \frac{y-3}{2} = \frac{x-2}{-1}$$

$\therefore -1(y + 3) = 2(x - 2)$

$\therefore\ -y + 3 = 2x — 4$

$\therefore\ 2x + y = 7$

Equation of median CF is

$$\frac{y-6}{\frac{7}{2}-6}=\frac{x-1}{\frac{3}{2}-1}$$

$$\therefore \frac{y-6}{-\frac{5}{2}}=\frac{x-1}{\frac{1}{2}}$$

$$\therefore \frac{y-6}{-5}=\frac{x-1}{1}$$

$\therefore y - 6 = -5(x - 1)$

$\therefore y - 6 = -5 + 5$

$\therefore 5x + y - 11 = 0.$

Miscellaneous Exercise 5 | Q 15.3 | Page 80

The vertices of a triangle are A (1, 4), B (2, 3) and C (1, 6). Find equations of Perpendicular bisectors of sides

SOLUTION

Vertices of ΔABC are A(1, 4), B(2, 3) and C(1, 6)

$\therefore$ Slope of perpendicular bisector of BC is $\frac{1}{3}$ and the lines passes through $\left(\frac{3}{2}, \frac{9}{2}\right)$.

$\therefore$ Equation of the perpendicular bisector of side BC is

$$\left(y-\frac{9}{2}\right)=\frac{1}{3}\left(x-\frac{3}{2}\right)$$

$$\therefore \frac{2y-9}{2}=\frac{1}{3}\left(\frac{2x-3}{2}\right)$$

$\therefore 3(2y - 9) = (2x -3)$

$\therefore 2x - 6y + 24 = 0$

$\therefore x - 3y + 12 = 0$

Since, both the points A and C have same x co-ordinates i.e. 1

$\therefore$ the points A and C lie on the line x = 1.

AC is parallel to Y-axis and therefore, perpendicular bisector of side AC is parallel to X-axis.

Since, the perpendicular bisector of side AC passes through E(1, 5).

$\therefore$ the equation of perpendicular bisector of side AC is y = 5.

Slope of side AB = $\left(\frac{3-4}{2-1}\right) = -1$

$\therefore$ Slope of perpendicular bisector of AB is 1 and the line passes through $\left(\frac{3}{2}, \frac{7}{2}\right)$.

$\therefore$ Equation of the perpendicular bisector of side AB is

$$\left(y-\frac{7}{2}\right) = 1\left(x-\frac{3}{2}\right)$$

$$\therefore \frac{2y-7}{2} = \frac{2x-3}{2}$$

$\therefore$ 2y – 7 = 2x – 3

$\therefore$ 2x – 2y + 4 = 0

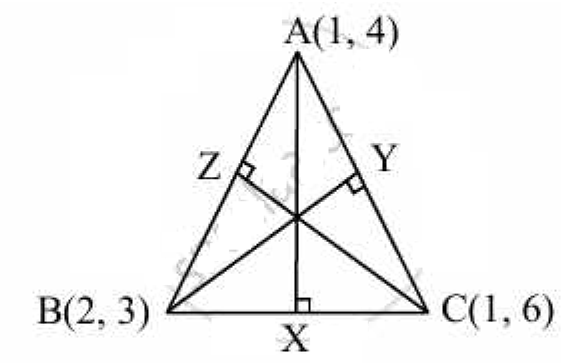
$\therefore$ x – y + 2 = 0.

Miscellaneous Exercise 5 | Q 15.4 | Page 80

The vertices of a triangle are A (1, 4), B (2, 3) and C (1, 6). Find equations of altitudes of ΔABC

SOLUTION

Vertices of ΔABC are A(1, 4), B(2, 3) and C(1, 6)



Let AX, BY and CZ be the altitudes through the vertices A, B and C respectively of ΔABC.

Slope of BC = – 3

$\therefore$ slope of AX = $\frac{1}{3}$ ...[$\because$ AX $\perp$ BC]

Since, altitude AX passes through (1, 4) and has slope $\frac{1}{3}$

$\therefore$ equation of altitude AX is

$y - 4 = \frac{1}{3}(x - 1)$

$\therefore 3y - 12 = x - 1$

$\therefore x - 3y + 11 = 0$

Since, both the points A and C have same x co-ordinates i.e. 1
$\therefore$ the points A and C lie on the line x = 1.

AC is parallel to Y-axis and therefore, altitude
BY is parallel to X-axis.
Since, the altitude BY passes through B(2, 3).

$\therefore$ the equation of altitude BY is y = 3.
Also, slope of AB = – 1
$\therefore$ slope of CZ = 1 ...[$\because$ CZ $\perp$ AB]

Since, altitude CZ passes through (1, 6) and has slope 1
$\therefore$ equation of altitude CZ is
$\therefore y - 6 = 1(x - 1)$
$\therefore x - y + 5 = 0.$